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COUPLED PROBLEMS OF THE OPTIMIZATION OF THE ELASTO-PLASTIC DEFORMATION OF METALS†

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The mathematical formulation of a coupled optimization problem, which includes several optimal problems which are inter-related via the optimization parameters and functions of state, is analysed. The conditions for the existence of a solution are formulated and a technique for reducing the problem, to a sequence of optimal problems with unknown functions in the optimality and bounding criteria is described. A special minimizing sequence is constructed and its convergence conditions are derived. An example is given to demonstrate the method. © 1999 Elsevier Science Ltd. All rights reserved.

The need to solve various inter-related problems arises, for example, when investigating the multi-transition thermomechanical processing of metals or the optimal design of structures to take account of their manufacturing technology. These problems are solved separately, without reference to one another. For example, there have been a large number of investigations on optimizing the distribution of the mechanical parameters of anisotropy or inhomogeneity of a material in the volume of a structure to give it minimum weight or maximum strength ([1–3], etc.), but they do not consider the choice of manufacturing technology which gives optimum usable structural properties. Original methods for the optimal control of temperature fields, stresses, deformations and displacements in the technology of material processing have been described in a number of papers ([4–6] among others), but how to choose optimal distributions on the basis of the conditions under which the structure is used is not considered. It is only possible to justify such a choice by the simultaneous solution of problems of optimal design of the structure and optimal control of its manufacturing technology.

There have been a small number of investigations in which specific manufacturing processes are analysed in this way (the reinforcement of structures of composite materials, for example). However, this does not enable different technological processes to be compared. A broader optimization is made more difficult by the lack of a sufficiently general technique for solving coupled optimization problems and by the complexity of the boundary-value problem of thermal elastoplasticity or creep, which describes the behaviour of a material during its manufacture.

Note that thermomechanical treatment is usually associated with large temperature and stress gradients and involves complex loading conditions and large deformations. The corresponding problems of thermal elastoplasticity are consequently time-consuming. The use of high-speed computers the development of efficient numerical solutions and the construction of a modern theory of constitutive relations ([7–9], etc.) has reduced the computer time needed considerably, making it feasible to attempt a statement and solution of coupled optimization problems.

1. AN EXAMPLE OF THE STATEMENT OF A COUPLED OPTIMIZATION PROBLEM

We will consider the formulation of the coupled optimization problem using the example of the optimization of the electrical forging of parts of axisymmetric shape. This involves two successive stages (Fig. 1): the piece is heated by passing an electric current through it and then immediately deformed by heating to obtain a piece of the required shape.

It has been found experimentally that the maximum ductility of the metal of a blank depends on many parameters, the main ones being the temperature θ, the intensity of the deformation rates H_u^z and the stressed state indicator and the stressed state indicator σ_c/T (σ_c is the average stress and T is the shear stress intensity). The problem is to choose the process parameters (the current distribution function over time, the heating time, the punch pressure, etc.) for which maximum use is made of the ductility of the metal. It is better to split the problem into two inter-related optimal problems: first, to optimize

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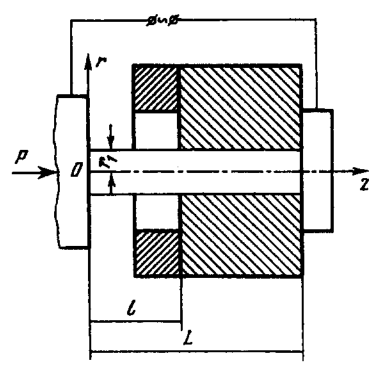

Fig. 1.

the heating of the blank to obtain a near-optimal temperature distribution and then, by controlling the pressure on the punch with the known heating temperature of the blank, to find an approximation to the optimum stress–strain state relative to some norm or other. The reason why these problems are so difficult is that they require the repeated solution of non-linear and time-dependent problems of electrothermal conduction and thermal elastoplasticity.

Using the technique described in [9, 10], we have constructed a mathematical model of the process which takes into account the non-linear distribution of the heat sources when a current is passed through the blank, complex loading and large plastic deformations of the metal. The agreement between the calculated and experimental results is good. We have established that the main factors associated with the failure of the metal for a given process are the temperature to which the blank is heated and the deformation rate. These factors can be monitored using the current strength function $I_0(t)$, $t \in [0, t_1]$, where t_1 is the time during which the blank is heated, and the change of pressure on the punch $P(t)$, $t \in [t_1, t_2]$, where $(t_2 - t_1)$ is the time during which the blank is deformed.

We thus obtain the following statement of a biconnected optimization problem: it is required to find controls from the class of piecewise-continuous functions $h_1(\cdot) = I_0(\cdot) \in \text{pc}[0, t_1]$ and $h_2(\cdot) = P(\cdot) \in \text{pc}[t_1, t_2]$ which give a minimum value to the following functions

$$J_{10}(h_1) = \int_0^L [\theta(t_1, r_1, z; h_1) - \bar{\theta}(z; H_u^z, \sigma_c / T)]^2 dz \to \inf_{h_1} \tag{1.1}$$

$$J_{20}(h_2) = \int_{t_1}^{t_2} \left[\max_{r,z \in \Omega_2^t} (H_u^z(t, r, z; h_2)) - \overline{H}_u^z(\tilde{\theta}, \sigma_c / T) \right]^2 dt \to \inf_{h_2} \tag{1.2}$$

with constraints in the form of equations

$$F_1(t, r, z, h_1, u_1, P_1) = 0, \quad t \in [0, t_1]; \quad r, z \in \overline{\Omega}_1^t \tag{1.3}$$

$$F_2(t, r, z, h_2, u_2, u_1(t_1, r, z), P_2) = 0, \quad t \in [t_1, t_2]; \quad r, z \in \overline{\Omega}_2^t \tag{1.4}$$

and inequalities

$$0 \leq h_1(t) \leq \bar{I}, \quad t \in [0, t_1] \tag{1.5}$$

$$0 \leq h_2(t) \leq \overline{P}, \quad t \in [t_1, t_2] \tag{1.6}$$

It is assumed here that

$$\bar{\theta}(z) = \begin{cases} \bar{\theta}_1(H_u^z, \sigma_c / T), z \in [0, l] \\ \bar{\theta}_2 = 50°\text{C}, z \in [l, L] \end{cases}, \quad \tilde{\theta} = \frac{1}{l} \int_0^l \theta(t_1, r_1, z) dz$$

and the functions $\bar{\theta}_1(H_u^z, \sigma_c/T)$ and $H_u^z(\bar{\theta}, \sigma_c/T)$ for which the two optimal problems are inter-connected are chose from experimental curves of ductility for the given material [11]. The temperature $\bar{\theta}_1$ corresponds to the maximum ductility of the metal for fixed values H_u^z and σ_c/T. The quantities u_1 and u_2 are taken as vector-functions of the state of the body, and P_1 and P_2 are taken as functions which specify external actions on the body during heating and deformation, respectively, $\bar{\Omega}_i^t$ ($i = 1, 2$) are the regions that the metal occupies during heating and deformation, $\bar{I}$ is the limit on the current $\bar{P}$ strength and is the limit of the rate of change of pressure on the punch.

Operator equations (1.3) and (1.4) are here the equations of time-dependent boundary-value problems of electrothermal conduction and thermal elastoplasticity. Solving these problems is computationally intensive. We will therefore first find a qualitative solution of optimal problems (1.1), (1.3), (1.5) and (1.2), (1.4), (1.6) with certain additional assumptions. Since the diameter of the body is much less than its length, with some confidence we can neglect the change of temperature over the radius. Then the constraint in the form of an equality for electrical heating optimization is the heat conduction equation with an internal source

$$c\gamma\frac{\partial\theta}{\partial t}=\lambda\frac{\partial^2\theta}{\partial z^2}+q(z,t),\quad z\in[0,L],\quad t\in[0,t_1] \tag{1.7}$$

$$q(z,t)=\begin{cases}I_0^2(t)R, & 0\leq z\leq l\\ 0, & l\leq z\leq L\end{cases}$$

where R is the resistance of the blank and c, λ and γ are the heat capacity, thermal conductivity and specific gravity of the material of the blank.

Note that the model used here is of a conductor in a conducting matrix. It is assumed that there is a current through the matrix in the section $z \in [l, L]$ and through the conductor in the section $z \in [0, l]$. The boundary conditions of the heat conduction problem are

$$\theta(z,0)=\theta_0,\quad z\in[0,L] \tag{1.8}$$

$$\partial\theta(z,t)/\partial z\big|_{z=0}=\partial\theta(z,t)/\partial z\big|_{z=L}=0,\quad t\in[0,t_1]$$

If the function $\bar{\theta}_1(H_u^z, \sigma_c/T)$ is assumed given, problem (1.1), (1.5), (1.7), (1.8) can be solved as follows.

We define the operator B: $L_2(0, t_1) \to L_2(0, L)$ by $(Bh_1^2)(z) = \theta(z, t_1) - \theta_0$. A solution of boundary-value problem (1.7), (1.8) is found using Fourier's method, which gives

$$\theta(z,t)=\theta_0+\frac{R}{c\gamma}\left[\frac{l}{L}\int_0^t h_1^2(\tau)d\tau+\right. \tag{1.9}$$

$$\left.+\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{k}\sin\left(k\pi\frac{l}{L}\right)\cos\left(k\pi\frac{z}{L}\right)\int_0^t h_1^2(\tau)\exp\left[\frac{\lambda}{c\gamma}\left(\frac{k\pi}{L}\right)^2(\tau-t)\right]d\tau\right]$$

It has been shown [12] that series (1.9) converges and its sum θ is a unique solution of problem (1.7), (1.8) almost everywhere (since the function q is discontinuous).

Functional (1.1) can then be rewritten in the form

$$J_{10}(h_1)=\int_0^L((Bh_1^2)(z)-g(z))^2dz=\left\|Bh_1^2-g\right\|^2,\quad g(z)=\bar{\theta}(z)-\theta_0 \tag{1.10}$$

It follows from (1.10) that $J_{10}(h_1)$ is Fréchet differentiable over the whole space $L_2(0, t_1)$, that is

$$\forall h\in L_2(0,t_1)\ \langle J_{10}(h_1),h\rangle=2(B^*(Bh_1^2-g),h)$$

where the operator B^*: $L_2(0, L) \to L_2(0, t_2)$ is conjugate to B.

It has been shown [12] that if $h_1 \in L_2(0, t_1)$, then $J_{10}(h_1)$ has no minimum on $L_2(0, t_1)$.

If $h_1 \in L_2(0, t_1)$, where $J_{10}(h_1)$ is a bounded closed and convex set in $L_2(0, t_1)$, then $B(H_1)$ is a convex compactum. Thus there is a function $h_1^* \in H_1$ which minimizes J_{10} on H_1. However, in that case the solution might not be unique. But if H_1 is chosen to be a convex closed set in the space H_{1m} of step functions

$$h_1(\tau)=a_i,\quad i=1,\ldots,m;\quad \tau_{i-1}\leq\tau\leq\tau_i;\quad 0=\tau_0<\tau_1<\ldots<\tau_m=t_1$$

for a fixed partition of the segment $[0, t_1]$, then the operator $B: H_{1m} \to L_2(0, L)$ is injective, and so the solution is unique.

Since we have the expression for the gradient $J'_{10}(h_1)$ in explicit form, we can obtain a numerical solution of the original optimal problem, using the gradient projection method for example.

However, the only function in the class of step functions which gives a simple realization of an actual technological process is discontinuous and periodic. For this reason we shall seek a solution of problem (1.1), (1.3), (1.5) in the above form. It is then easy to reduce the optimization (with no additional assumptions) to a problem of non-linear programming, with the current intensity during heating and the ratio of the heating time to the pause time as the optimization parameters. Figure 2 shows the solution for one of the blanks. The discontinuous heating regime (curve 2 in the upper right of Fig. 2) clearly gives a better temperature distribution over the surface of the blank (curve 2 on the left of Fig. 2) than the existing heating regime (curve 1 on the left of Fig. 2).

We will now consider the qualitative solution of the second optimal problem (1.2), (1.4), (1.6). On the basis of prior experimental and theoretical research we will make the following assumptions. We consider deformation of only the forged part of the blank l and assume the process to be isothermal. We take the model of an incompressible linearly viscous medium as a model of the material. Then constraints (1.4) can be written in the form of the differential relations

$$\sigma(t) = \mu(\tilde{\theta})\dot{\varepsilon}(t), \quad \dot{\sigma}(t) = h_2(t); \quad t \in [t_1, t_2] \tag{1.11}$$

with the corresponding boundary conditions

$$\sigma(t_1) = \varepsilon(t_1) = \dot{\varepsilon}(t_1) = 0, \quad \varepsilon(t_2) = \bar{\varepsilon} \tag{1.12}$$

where $\mu(\tilde{\theta})$ is the coefficient of viscosity of the material, which depends on the temperature to which the blank is heated and $\bar{\varepsilon}$ is the given longitudinal deformation. Note that, due to the incompressibility of the material and the axial symmetry of the blank, the intensity of the deformation rates H_u^z is equal to the longitudinal deformation rate $\bar{\varepsilon}$. Then the functional $J_{20}(h_2)$ can be written in the simplified form

$$J_{20}(h_2) = \int_{t_1}^{t_2} (\dot{\varepsilon}(t) - \bar{\dot{\varepsilon}})^2 dt$$

In addition, we assume that $\dot{P}(t) = \dot{\sigma}(t)$, where σ is the longitudinal stress.

Choosing the phase variable $x(\cdot) = \dot{\varepsilon}(\cdot) \in KC^1[t_1, t_2]$, we apply Pontryagin's maximum principle to the given optimal control problem. The Lagrange function has the form

$$\mathfrak{L} = \int_{t_1}^{t_2} \left[\lambda_0(x(t) - \bar{\dot{\varepsilon}})^2 + \lambda_1 x(t) + p(t)(\dot{x}(t) - \mu^{-1} h_2(t))\right] dt - \lambda_1 \bar{\varepsilon}$$

where λ_1, λ_2 are Lagrange multipliers, $p(\cdot)$ is the adjoint function, and t_2 is a variable quantity.

Then the required optimality conditions have the form

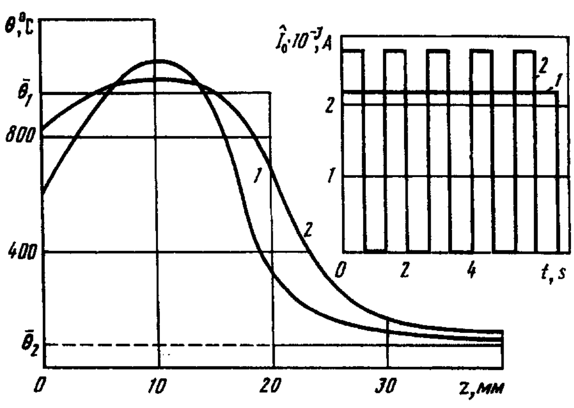

Fig. 2.

$$\dot{p}(t)=\lambda_0 2(x(t)-\bar{\varepsilon})+\lambda_1,\quad p(t_2)=0 \tag{1.13}$$

$$\max_{h_2(t)\leqslant P}(p(t)\mu^{-1}h_2(t))=p(t)\mu^{-1}\hat{h}_2,\quad \lambda_0(x(t_2)-\bar{\varepsilon})^2+\lambda_1 x(t_2)=0$$

It follows from the optimality condition for h_2 that

$$\hat{h}_2(t)=\bar{P}\,\mathrm{sign}(p(t)),\quad p(t)\neq 0 \tag{1.14}$$

It can be shown that $\lambda_0 \neq 0$. Then we shall take $\lambda_0 = 1$. Investigating the function $p(\cdot)$ using conditions (1.13) and differential relations (1.11), we can show that $p(\cdot)$ is a continuous function, with $p(t_1) > 0$.

It follows from (1.11), (1.12) and (1.14) that

$$\hat{h}_2(t)=\bar{P},\quad \hat{x}(t)=\mu^{-1}\bar{P}(t-t_1),\quad t\in[t_1,\tau],\quad \tau=\min\{t\in[t_1,t_2]/\,p(t)\leqslant 0\}$$

We will consider the following possibilities.

1. Let $\tau = t_2$. Then $\hat{x}(t_2) = \mu^{-1}\bar{P}(t - t_1)$; $\hat{t}_2 = \sqrt{2\mu\bar{\varepsilon}/\bar{P}}$.

We obtain the solution

$$\hat{h}_2(t)=\bar{P};\quad \hat{x}(t)=\mu^{-1}\bar{P}(t-t_1),\quad t\in[t_1,\hat{t}_2] \tag{1.15}$$

which holds when $\bar{P}\leqslant \dot{\bar{\varepsilon}}^2\mu/(2\bar{\varepsilon})$.

2. Let $\tau < t_2$. Then $p(\tau) = 0$. Hence $\tau = \dot{\bar{\varepsilon}}\mu/\bar{P}$

It can be shown that $p(t) = 0$, $t \in [\tau, t_2]$. Then for $\bar{P} > \dot{\bar{\varepsilon}}^2\mu/(2\bar{\varepsilon})$ we have the optimal solution

$$\hat{h}_2(t)=\begin{cases}\bar{P}, & t\in[t_1,\tau)\\ 0, & t\in[\tau,\hat{t}_2]\end{cases},\quad \hat{x}(t)=\begin{cases}\mu^{-1}\bar{P}(t-t_1), & t\in[t_1,\tau)\\ \bar{\varepsilon}, & t\in[\tau,\hat{t}_2]\end{cases} \tag{1.16}$$

$$\hat{t}_2=\dot{\bar{\varepsilon}}\mu/(2\bar{P})+\bar{\varepsilon}/\dot{\bar{\varepsilon}}$$

The fact that the admissible extrema (1.15) and (1.16) are unique means that the solution of problem (1.2), (1.4), (1.6) in the given formulation is unique. Also it can be shown that the extremum for the functional J_{20} given by solutions (1.15) and (1.16) is global.

Figure 3 shows solution (1.16). We see that the control h_2 must have a witch at a certain point τ, which depends on the temperature to which the blank is heated and the mechanical properties of its material (Fig. 3a). In this case the intensity of the deformation rates is closest to the given value throughout the entire deformation process (Fig. 3b), thereby reducing the possibility of metal failure.

Note that the first and second problems of optimal control here are coupled. The first problem contains the quantity $\bar{\theta}_1$, which depends on the solution of the second problem $H_u^z(t)$, and the solution of the second depends on the quantity $\tilde{\theta}$, which is found from the solution of the first. Thus these problems must be solved by iteration. We found that the convergence of the iteration depends to a large extent on the form of the coupling functions $\bar{\theta}_1(H_u^z)$ and $H_u^z(\theta)$. If these are smooth enough, convergence is achieved after 3–4 iterations.

The coupled problem of [13] was solved to obtain the optimal heating and deformation regimes whereby, as Fig. 4 shows, much greater use is made of the plastic properties of the material and exhaustion of the ductility reserve Ψ [11], which was taken as a criterion of failure of the metal (Ψ = 1), is reduced. In Fig. 4 curve 1 corresponds to the build-up of damage in the metal under the existing heating and deformation regimes, curve 2 corresponds to optimal deformation, curve 3 corresponds to optimal heating, and curve 4 corresponds to the two together. Under optimal heating and deformation conditions, exhaustion of the ductility reserve of the metal can be reduced by more than 20%. This was confirmed by experimental data obtained under real conditions. Nomograms of rational forging programmes constructed on the basis of these results for a broad class of blanks of typical sizes have been introduced in a number if industrial enterprises [14].

2. STATEMENT OF THE BI-OPTIMIZATION PROBLEM IN THE GENERAL CASE

Suppose a given body occupies the bounded three-dimensional region Ω_1^t with boundary Γ_1^t in the time interval $[t_0, t_1]$, and the region Ω_2^t with boundary Γ_2^t in the time interval $[t_1, t_2]$. It will be assumed that $\overline{\Omega}_2^{t_1} = \overline{\Omega}_1^{t_1}$, where $\overline{\Omega}_n^{t_1} = \Omega_n^{t_1} \cup \Gamma_n^{t_1}$ (here and below, unless otherwise stated, $n = 1, 2$). We introduce the vector functions of the state of the body $\mathbf{u}_1(\cdot, \cdot) \in w_2^{0,1}([t_0, t_1] \times \Omega_1^t)$ and $\mathbf{u}_2(\cdot, \cdot) \in w_2^{0,1}([t_1, t_2] \times \overline{\Omega}_2^t)$, where $w_2^{0,1}([t_{n-1}, tn] \times \overline{\Omega}_n^t)$ are Sobolev spaces. In the problems considered here, these can be taken as functions of the displacement, strain, stress, temperature, etc. which characterize the state of the deformed body at a given point and a given time. Let $\mathbf{h}_n(\cdot, \cdot) \in \mathrm{pc}([t_{n-1}, tn] \times \overline{\Omega}_n^t)$ denote the vector functions of the optimization parameters in the respective spaces of the piecewise-continuous functions. We shall consider functionals of the form

$$J_{1i}(\mathbf{h}_1) = J_{1i}(\mathbf{h}_1, \mathbf{u}_1(h_1), \varphi_1(\mathbf{h}_2, \mathbf{u}_2)), \quad i = 0, \ldots, m_1$$

$$J_{2i}(\mathbf{h}_2) = J_{2i}(\mathbf{h}_2, \mathbf{u}_2(\mathbf{h}_2), \varphi_2(\mathbf{h}_1, \mathbf{t}_1, \cdot))), \quad i = 0, \ldots, m_2$$

where φ_1, φ_2 are the continuous operators due to which the two optimal problems are inter-connected.

The operator φ_2 allows for the entire history of deformation in the previous state (deformation hardening of the material, residual stresses, etc.) and φ_1 takes into account the influence of the functions of state and optimization parameters of the second stage of deformation on the first. The dependence on φ_1 in the problem as formulated is unconventional, since at any instant of time the solution depends on that at the next instant, so that the "actual" value depends not only on the "past", but also on the "future", which has never happened before. This is the main novelty of the statement of the bi-optimization problem and it is this which makes it difficult to solve. Furthermore, $\mathbf{u}_1(\mathbf{h}_1)$ and $\mathbf{u}_2(\mathbf{h}_2)$ are usually given in terms of a system of non-linear differential or integro-differential equations rather than in explicit form.

Let the relation between the functions of state and optimization parameters be given by a system of equations which can be written in operator form as follows

$$F_1(t, \mathbf{x}, \mathbf{h}_1, \mathbf{u}_1, \varphi_1(\mathbf{h}_2, \mathbf{u}_2), P_1) = 0, \quad t \in [t_0, t_1], \quad \mathbf{x} \in \overline{\Omega}_1^t$$

$$F_2(t, \mathbf{x}, \mathbf{h}_2, \mathbf{u}_2, \varphi_2(\mathbf{u}_1(t_1, \cdot)), P_2) = 0, \quad t \in [t_1, t_2], \quad \mathbf{x} \in \overline{\Omega}_2^t$$

where the operators F_n describe the behaviour of the body during deformation, the functions $P_n(\cdot, \cdot) \in L_2([t_{n-1}, tn] \times \overline{\Omega}_n^t)$ set the loads on the body during deformation and belong to the Lebesgue space L_2 and $\mathbf{x}$ is the space coordinate vector. In the case under consideration, the above system of equations are the equations of the boundary-value problem of thermal elastoplasticity.

Deformation of a body is always subject to technological and design constraints. These can be written in the form of the inequalities

$$f_{nj}(t, \mathbf{x}, \mathbf{h}_n, \mathbf{u}_n, J_{ni}) \leqslant 0, \quad t \in [t_{n-1}, t_n], \quad \mathbf{x} \in \overline{\Omega}_n^t; \quad i = 1, \ldots, m_n; \quad j = 1, \ldots, k_n$$

where f_{nj} are given continuous functions. If the optimality criteria of each stage of deformation are taken to be the conditions for the minimization of functionals J_{10} and J_{20}, respectively, then the bi-optimization problem can be stated as follows: it is required to find functions $\hat{\mathbf{h}}_1$, $\hat{\mathbf{h}}_2$ such that

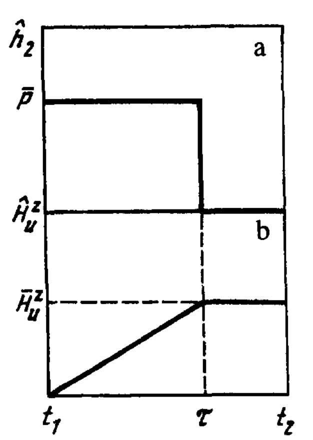

Fig. 3.

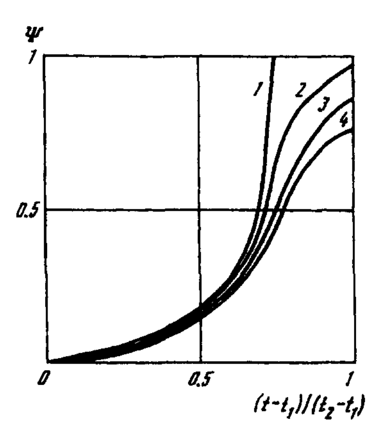

Fig. 4.

$$J_{n0}(\mathbf{h}_n) \to \inf, \ n = 1,2 \tag{2.1}$$

with equation constraints

$$F_n(t, \mathbf{x}, \mathbf{h}_n, \mathbf{u}_n, \varphi_n, P_n) = 0, \ t \in [t_{n-1}, t_n], \ \mathbf{x} \in \overline{\Omega}_n^t \tag{2.2}$$

and inequality constraints

$$f_{nj}(t, \mathbf{x}, \mathbf{h}_n, \mathbf{u}_n, J_{ni}) \leq 0, \ t \in [t_{n-1}, t_n], \ \mathbf{x} \in \overline{\Omega}_n^t; \ i = 1, ..., m_n; \ j = 1, ..., k_n \tag{2.3}$$

This is not a two-criteria problem in the ordinary sense. The regions of definition of functionals (2.1) are separate in time, and so there is no point in introducing the concept of Pareto optimization [15]. In this case we consider two optimization parameters and functions of state of one problem on the solution of the other. A new approach to optimization of this type is this required.

We will consider the conditions for a solution of problem (2.1)–(2.3) to exist. To do so, we introduce the auxiliary sets U_1 and U_2 in which constraints (2.2) are satisfied, and sets H_1 and H_2 in which constraints (2.3) are satisfied, respectively. We have the following theorem.

Theorem. Suppose $\mathbf{u}_n \in U_n$ exist such that the sets H_n are non-empty, bounded and weakly closed; let the conditions of continuity of the mapping $H_1 \times U_1 \to R$ given by the functionals J_{1i} $(i, 0, \ldots, m_1)$, $\forall \mathbf{h}_2 \in H_2$, $\forall \mathbf{u}_2 \in U_2$, and the mapping $H_2 \times U_2 \to R$, given by the functionals J_{2i} $(i = 0, \ldots, m_2)$, $\forall$ $\mathbf{u}_1(t_1, \cdot)$ $\in U_1$, hold; let the functionals J_{m0} be convex with respect to $\mathbf{h}_n$ and φ_n, $\forall$ $\mathbf{u}_n \in U_n$, respectively, and let the functions φ_n, f_{nj} $(j = 1, \ldots, k_n)$ be continuous with respect to each argument.

Then, (1) the solution of problem (2.1)–(2.3) $\hat{\mathbf{h}}_n$ exists; (2) sequences $\mathbf{h}_n^{(k)}$ $(k = 1, 2, \ldots)$, satisfying conditions

$$J_{n0}(\hat{\mathbf{h}}_n) = \lim_{k \to \infty} J_{n0}(\mathbf{h}_n^{(k)}) = \inf J_{n0}(\mathbf{h}_n), \ \mathbf{h}_n \in H_n$$

can be found and $\| \mathbf{h}_n^{(k)} - \hat{\mathbf{h}}_n \|_c \to 0$.

Remarks. 1. The condition of quasi-regularity of the functionals J_{n0} (convexity with respect to $\mathbf{h}_n$, φ_n), can be replaced by their weak semi-continuity with respect to $\mathbf{h}_n$ and φ_n [16].

2. We will not give the proof of the theorem here, but it is easily obtained from the theorems and proofs given in [16, 17].

The theorem suggests a general method of solving coupled optimization problems, in which each is split into a number of individual optimal problems which are solved in succession. A special iterative procedure can be constructed for this purpose. We begin by considering problem (2.1)–(2.3) $(n = 2)$ for a given function $\hat{\mathbf{u}}_1^{(0)}(t_1, \cdot)$ (usually $\hat{\mathbf{u}}_1^{(0)}(t_1, \cdot) = 0$). This leads to the classical statement of the given optimization. By using a known optimization method to solve it, we find $\hat{\mathbf{h}}_2^{(0)}$, and then use F_2 to find $\hat{\mathbf{u}}_2^{(0)}\hat{\mathbf{h}}_2{}^{(0)}$. (We recall that $\hat{\mathbf{u}}_2^{(0)}\hat{\mathbf{h}}_2^{(0)}$ denotes the solution of the boundary-value elastoplasticity problem (2.2) $(n = 2)$ for a certain distribution of the optimization parameters $\hat{\mathbf{h}}_2^{(0)}$). Further, we solve optimization problem (2.1)–(2.3) $((n = 1))$ with fixed functions $\hat{\mathbf{u}}_2^{(0)}$ and $\hat{\mathbf{h}}_1^{(1)}$ and find $\hat{\mathbf{u}}_1^{(1)}$. We then refine the function $\hat{\mathbf{u}}_1^{(1)}(t_1, \cdot)$ and again solve problem (2.1)–(2.3) $((n = 2))$. The iteration is continued until the convergence conditions are satisfied.

This method of solving coupled problems is not unique. For example, one could construct special or generalized functionals which depend on all the optimization parameters, as in the theory of the solution of multicriterion problems [15]. However the possibility of using separate deformation stages, which is a promising feature of the technique. In some cases, moreover, the coupled optimization problem is made much simpler to solve because of the considerable simplification of the separate optimal problems [18].

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